Bayesian Network Example

By : **Peyman Navidi**

Bayesian Theorem

- Bayes' theory, constitutes the cornerstone of Bayesian learning
- This theory, possible calculate of posterior probability based on Initial probabilities

Likelihood

Prior probability

$$P(H|X) = \frac{P(X|H) \times P(H)}{P(X)}$$

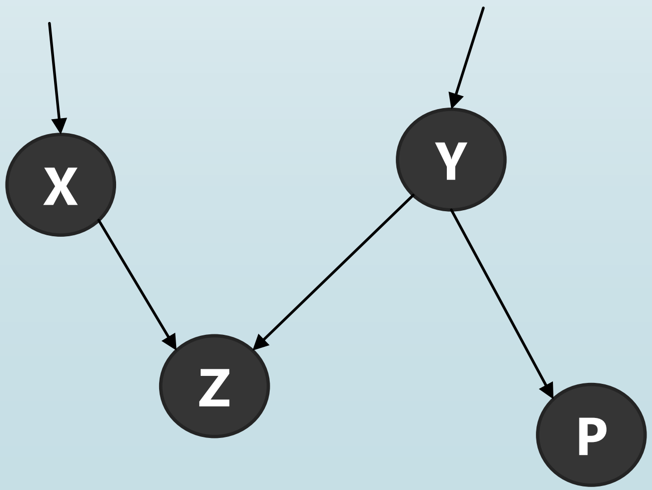
Posteriori probability

Evidence

posteriori = likelihood x prior / evidence

Bayesian Belief Networks

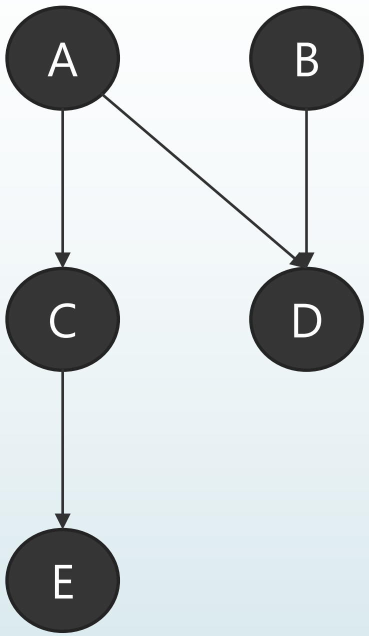
- Bayesian belief networks allow class conditional independencies between subsets of variables
- A graphical model of causal relationships
  - Represents dependency among the variables
  - Gives a specification of joint probability distribution
- The conditional probability table (CPT) for variables
- Derivation of the probability of a particular combination of values of X, from CPT



$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} P(x_i | Parents(Y_i))$$

- Nodes : random variables
- Links : dependency
- X and Y are the parents of Z, and Y is the parent of P
- No dependency between Z and P
- Has no loops or cycles

Example #1 of Bayesian Network



- Thus, the independence expressed in this Bayesian net are that :

A and **B** are (absolutely) independent.
C is independent of **B** given **A**.
D is independent of **C** given **A** and **B**.
E is independent of **A**, **B**, and **D** given **C**.

- Suppose that the net further records the following probabilities:

Prob(**A**=T) = **0.3**
Prob(**B**=T) = **0.6**
Prob(**C**=T|**A**=T) = **0.8**
Prob(**C**=T|**A**=F) = **0.4**
Prob(**D**=T|**A**=T,**B**=T) = **0.7**
Prob(**D**=T|**A**=T,**B**=F) = **0.8**
Prob(**D**=T|**A**=F,**B**=T) = **0.1**
Prob(**D**=T|**A**=F,**B**=F) = **0.2**
Prob(**E**=T|**C**=T) = **0.7**
Prob(**E**=T|**C**=F) = **0.2**

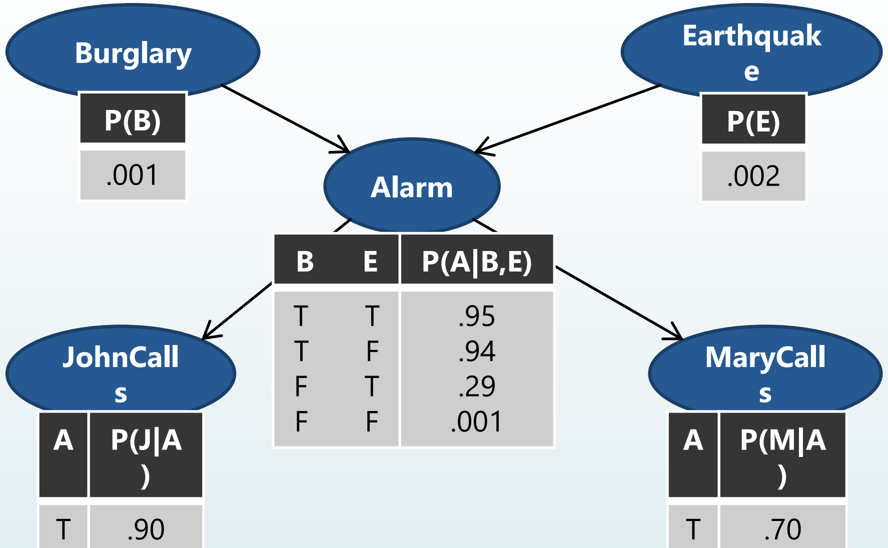
Prob(D=T) :

P(**D**=T) =
P(**D**=T,**A**=T,**B**=T) + P(**D**=T,**A**=T,**B**=F) +
P(**D**=T,**A**=F,**B**=T) + P(**D**=T,**A**=F,**B**=F) =
P(**D**=T|**A**=T,**B**=T) P(**A**=T,**B**=T) + P(**D**=T|**A**=T,**B**=F) P(**A**=T,**B**=F) +
P(**D**=T|**A**=F,**B**=T) P(**A**=F,**B**=T) + P(**D**=T|**A**=F,**B**=F) P(**A**=F,**B**=F) =
(since A and B are independent absolutely)
P(**D**=T|**A**=T,**B**=T) P(**A**=T) P(**B**=T) + P(**D**=T|**A**=T,**B**=F) P(**A**=T) P(**B**=F) +
P(**D**=T|**A**=F,**B**=T) P(**A**=F) P(**B**=T) + P(**D**=T|**A**=F,**B**=F) P(**A**=F) P(**B**=F) =

0.7×0.3×0.6 + 0.8×0.3×0.4 + 0.1×0.7×0.6 + 0.2×0.7×0.4 = **0.32**

Example #2 of Bayesian Network

Prob(**B**=T) = **0.001**
Prob(**E**=T) = **0.002**
Prob(**A**=T|**B**=T,**E**=T) = **0.95**
Prob(**A**=T|**B**=T,**E**=F) = **0.94**
Prob(**A**=T|**B**=F,**E**=T) = **0.29**
Prob(**A**=T|**B**=F,**E**=F) = **0.001**
Prob(**J**=T|**A**=T) = **0.90**
Prob(**J**=T|**A**=F) = **0.05**
Prob(**M**=T|**A**=T) = **0.70**
Prob(**M**=T|**A**=F) = **0.01**



Prob(B=T|A=F)
=**P(A=F|B=T) P(B=T) / P(A=F)**

P(A=F|B=T) :
= P(A=T,E=T|B=T) + P(A=F,E=F|B=T)
= P(A=F|E=T,B=T)P(E=T|B=T) + P(A=F|E=F,B=T)P(E=F|B=T)
= P(A=F|E=T,B=T)P(E=T) + P(A=F|E=F,B=T)P(E=F) = 0.05×0.002 + 0.06×0.998 = **0.056**

P(A=F) :
=P(A=F,B=T,E=T) + P(A=F,B=T,E=F) + P(A=F,B=F,E=T) + P(A=F,B=F,E=F)
=P(A=F|B=T,E=T)P(B=T,E=T)+P(A=F|B=T,E=F)P(B=T,E=F)+P(A=F|B=F,E=T)P(B=F,E=T)+P(A=F|B=F,E=F)P(B=F,E=F)
=P(A=F|B=T,E=T)P(B=T)P(E=T)+P(A=F|B=T,E=F)P(B=T)P(E=F)+P(A=F|B=F,E=T)P(B=F)P(E=T)+P(A=F|B=F,E=F)P(B=F)P(E=F)
=0.05×0.001×0.002 + 0.06×0.001×0.998 + 0.71×0.999×0.002 + 0.999×0.999×0.998 = **0.997**

Prob(B=T|A=F) = 0.056 × 0.001 / 0.997 = 0.000056

References

- **New York University Computer Science**

  http://cs.nyu.edu/faculty/davise/ai/bayesnet.html

- **Vancouver UBC University Computer Science**

  http://www.cs.ubc.ca/~murphyk/Bayes/bnintro.html